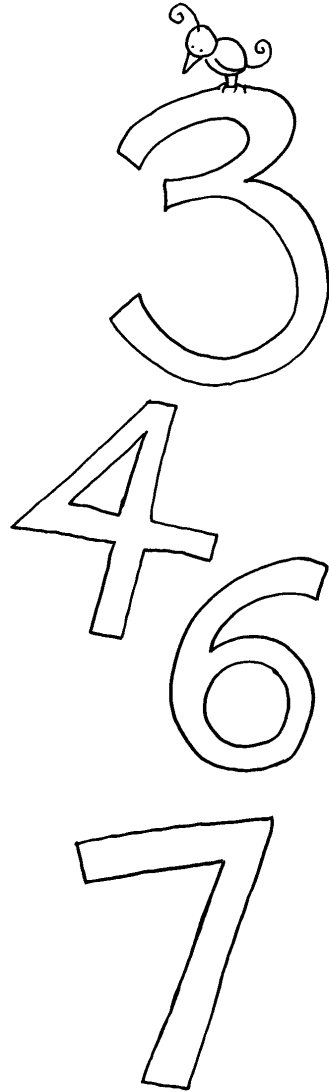


SOLVE  
THAT PROBLEM!

# Open-Ended Problem Solving

SOLVE  
THAT PROBLEM!



SOLVE  
THAT PROBLEM!

SOLVE  
THAT PROBLEM!

# Teaching Notes

## Open-Ended Problem Solving



Open-ended problems challenge a learner's thinking. In looking at open-ended problems, pupils explore problems that may be answered in a number of ways. However, these problems should always be accurately computed.

These problem solving activities are vitally important, and assist teachers to gauge the level at which their pupils are achieving, because pupils respond to challenges at their own level of development. The process that is used is more important than simply achieving the answer. When structuring problems, words such as create, make, design, investigate and explore should be used.

Ask pupils to develop their own open-ended problems. It is important that pupils develop their own open-ended problems and solutions as this involves them in developing their thinking and phrasing. If you are helping pupils solve problems they have set up, model your working as you solve them, as pupils will learn from your approach to their problems.

There are some specific skills and strategies that are useful when working with this approach.

### USING NUMBERED OR LABELLED COUNTERS

When problems become involved, using numbered or labelled counters can help pupils visualise a problem and its solutions. The counters can be easily manipulated and altered and other combinations found. Changes can easily be made if work is incorrect. When the correct solution is found, it can be written down.

### TRYING DIFFERENT COMBINATIONS OF NUMBERS

For example, if attempting to solve a problem in which you are to determine the largest product possible using five different numbers, start by using the largest three digit number and then the largest two digit number to see the different answers. (See also teaching example 1.)



Using the largest three-digit number:

$$\begin{array}{r} 987 \\ \times 65 \\ \hline 64155 \end{array}$$

Using the largest two-digit number:

$$\begin{array}{r} 642 \\ \times 98 \\ \hline 62916 \end{array}$$

### CONTINUE WORKING TO FIND AS MANY SOLUTIONS AS POSSIBLE

This involves pupils in working to find more than one answer, and manipulating figures in order to look at them differently. Pupils are thereby encouraged to become involved with genuine mathematical solutions.

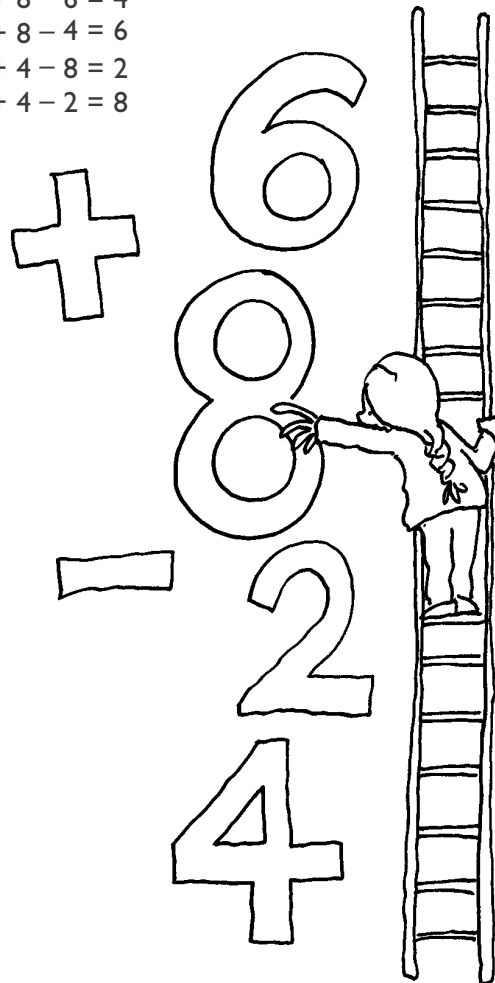
eg What can we do with 2 4 6 8 = + - ?

$$2 + 8 - 6 = 4$$

$$2 + 8 - 4 = 6$$

$$6 + 4 - 8 = 2$$

$$6 + 4 - 2 = 8$$



# Teaching Examples Open-Ended Problem Solving



## EXAMPLE 1

Explore different arrangements of the digits 2, 4, 6, 8 and 9 so that you create the largest product possible, using only one multiplication sign ( $\times$ ). Each numeral must be used, and can be used only once.

### Understanding the problem

#### WHAT DO WE KNOW?

There are different arrangements of digits, one of which will enable us to reach the highest possible product.

#### WHAT DO WE NEED TO FIND OUT?

Questioning: What are the digits? How should they be arranged?

#### COMMUNICATING A SOLUTION

Begin by using the largest three digit number you can make:

$$\begin{array}{r} 986 \\ \times 42 \\ \hline 41412 \end{array}$$

Now try using the largest two digit number.

$$\begin{array}{r} 642 \\ \times 98 \\ \hline 62916 \end{array}$$

As we multiply numbers together we see that  $800 \times 90$  or  $900 \times 80$  will give you 72 000.

So:

$$\begin{array}{r} 842 \\ \times 96 \\ \hline 80832 \end{array}$$

or:

$$\begin{array}{r} 962 \\ \times 84 \\ \hline 80808 \end{array}$$

While:

$$\begin{array}{r} 862 \\ \times 94 \\ \hline 81028 \end{array}$$

$$\begin{array}{r} 942 \\ \times 86 \\ \hline 81012 \end{array}$$

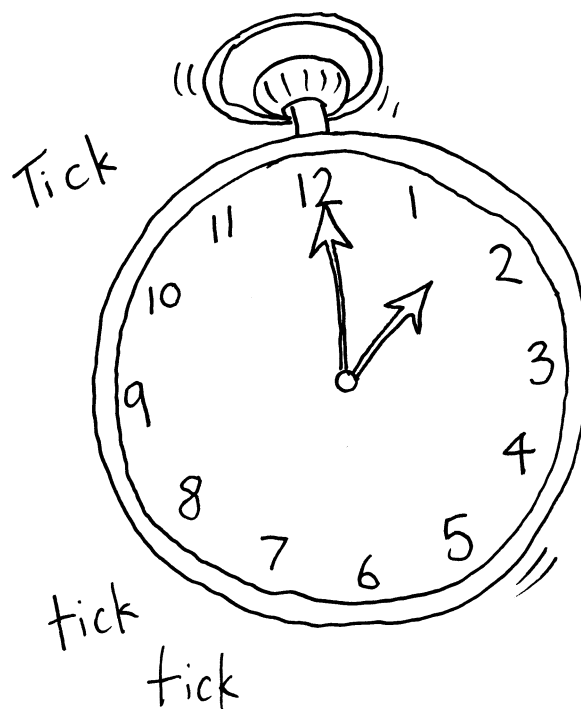
$862 \times 94$  resulted in the largest product.

### Reflecting and generalising

The process of trying one solution and then trying others allows pupils to become familiar with what happens when we multiply different combinations of numbers. This knowledge can then be applied to numerous other problems and will form part of their solutions.

### Extension

Repeat the example making the multiplication 3 digits  $\times$  3 digits. This will take a lot longer, so allow plenty of time.





## EXAMPLE 2

Investigate which combinations of the digits 3, 4, 6, and 7 will create addition problems that have a sum falling between 100 and 120.

### Understanding the problem

WHAT DO WE KNOW?

There are different combinations of digits that can be used in addition problems. These will enable us to reach the appropriate total.

WHAT DO WE NEED TO FIND OUT?

Questioning: What combination of digits total between 100 and 120?

### COMMUNICATING A SOLUTION

By trying different combinations the following were found to be suitable solutions.

$$\begin{array}{r} 43 \\ +76 \\ \hline 119 \end{array} \quad \begin{array}{r} 34 \\ +67 \\ \hline 101 \end{array} \quad \begin{array}{r} 43 \\ +67 \\ \hline 110 \end{array} \quad \begin{array}{r} 34 \\ +76 \\ \hline 110 \end{array}$$

$$\begin{array}{r} 37 \\ +64 \\ \hline 101 \end{array} \quad \begin{array}{r} 36 \\ +74 \\ \hline 110 \end{array}$$

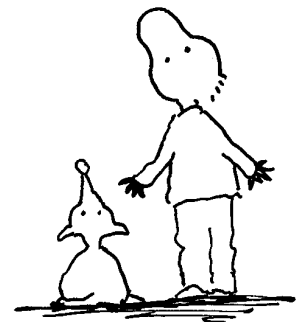
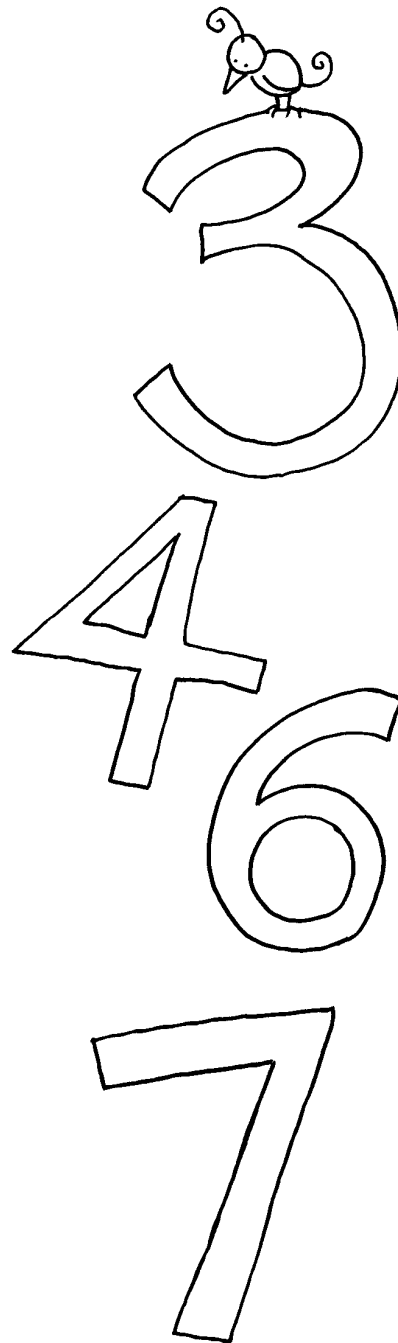
### Reflecting and generalising

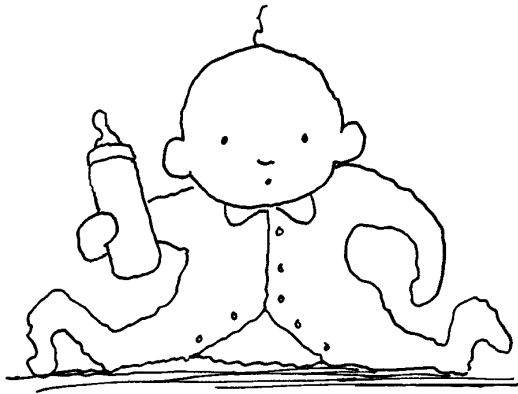
The process of trying one solution and then trying others allows us to become familiar with what happens when we add different combinations of numbers.

This knowledge can then be applied to numerous other problems and will form part of their solutions.

### Extension

The same problem can be given using subtraction, for example: using the digits 2, 4, 6, and 7 which combinations when subtracted will have a difference of between 20 and 30?





## EXAMPLE 3

There are three children. The product of their ages is 20. How old might they be? (A 'child' is counted as being under 18, and the ages in this problem are calculated in whole years.)

### Understanding the problem

#### WHAT DO WE KNOW?

There are three whole numbers. The three numbers are multiplied to make the product 20. There are different arrangements of digits that will enable us to reach the appropriate total.

#### WHAT DO WE NEED TO FIND OUT?

Questioning: How old are the children? Which numbers were multiplied together?



### Communicating a solution

There are groups of three numbers that can be multiplied together to make 20. (We are finding the factors of 20.)

1, 2, 10  
1, 4, 5  
2, 2, 5  
1, 1, 20

The answer 1, 1, 20 is not appropriate, as a 'child' must be under eighteen, so we are left with three answers, all of which could be correct.

### Reflecting and generalising

The process of finding a number of solutions or all the solutions encourages pupils to attempt different methods. Depending on their mathematical ability, they may or may not understand that they are looking at factors.

### Extension

What if there were four children and the product of their ages was 36? What is the lowest possible product of the ages of two adults? Of three adults?

# Copymaster Open-Ended Problem Solving



## ★ Understanding the problem

List what you know from reading the problem .....

## ★ What do you need to find out?

What questions do you have? What are you uncertain about? Is there any unfamiliar or unclear language? .....

## ★ Planning and communicating a solution

Start by reading the problem. Decide what you are being asked to do. Decide where to start. Try to work methodically, thinking all the way through one part at a time. Try to find as many solutions as possible. Have you found all the solutions? Can you see a pattern?

## ★ Reflecting and generalising

Did the strategy work as planned? Will you be able to apply this method of problem solving to other similar problems? Would a different method have worked better for you for this problem?

## ★ Extension

How can this strategy be applied to more complicated problems involving additional factors? .....

**Problem 55**

Shape & Space

Level  
**A**

Write everything that you know about a rectangle.

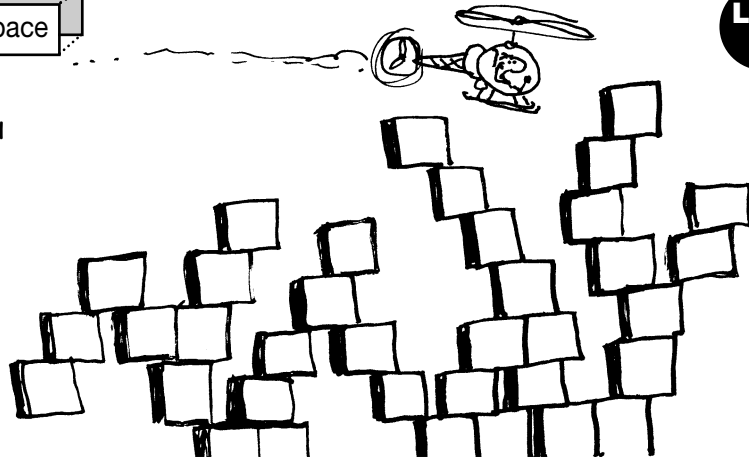


**Problem 56**

Shape & Space

Level  
**A**

Explore how many ways you can create a building using 36 cubes.

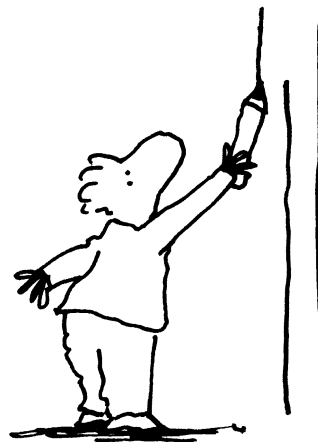


**Problem 57**

Measures

Level  
**A**

The total length of three lines is 16 cm. Each line is more than 3 cm long but less than 12 cm. How many different sets of lines can you draw that add up to this total length? (Only use whole centimetres.)



# PROBLEM SOLVING TASK CARDS -

## Open-Ended Problem Solving

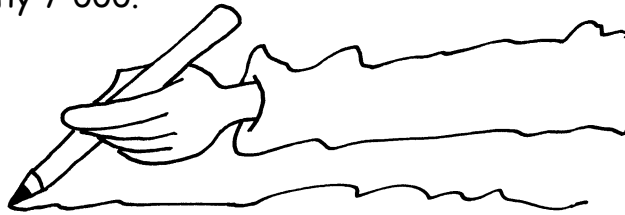
### Problem 58

Numbers **123**

Level  
**A**

Write five different digits in the boxes so that the product is as close to 7 000 as possible, but not exactly 7 000.

$$\begin{array}{r} \square \square \square \square \\ \times \square \\ \hline \end{array}$$

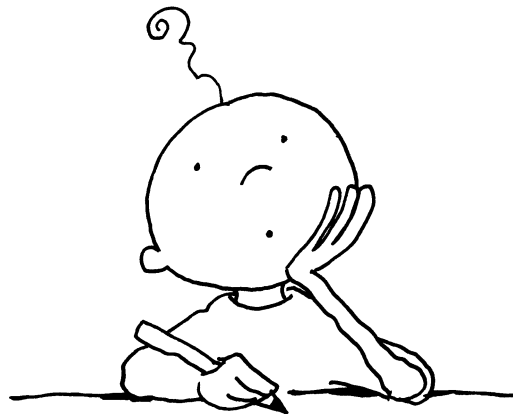


### Problem 59

Numbers **123**

Level  
**A**

How many equations can you make using the numbers 3, 2, 6, 1, and the symbols =, +, ÷? You can only use each numeral once in any equation, but you may use the signs more than once.



### Problem 60

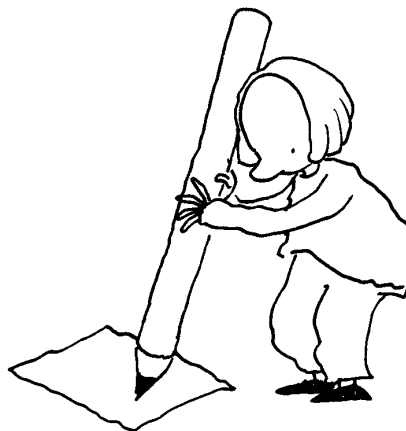
Numbers **123**

Level  
**A**

Fill in the boxes to make the answer true:

$$\begin{array}{r} 7 \square 9 \\ + \square 5 \square \\ \hline \square 3 4 5 \end{array}$$

Is there more than one solution?



# PROBLEM SOLVING TASK CARDS -

## Open-Ended Problem Solving

### Problem 61

Shape & Space

Level B

Create as many triangles as you can using five toothpicks. How many can you make with 7, 9, 11 or 13 toothpicks? How many can you make with 21? Is there a pattern? Draw them below as you make them.

What happens if there is an even number of toothpicks?



### Problem 62

Shape & Space

Level B

Take twelve 1 cm cubes. How many different rectangular prisms can you make using all twelve cubes? Draw them. Is the volume the same for each prism?

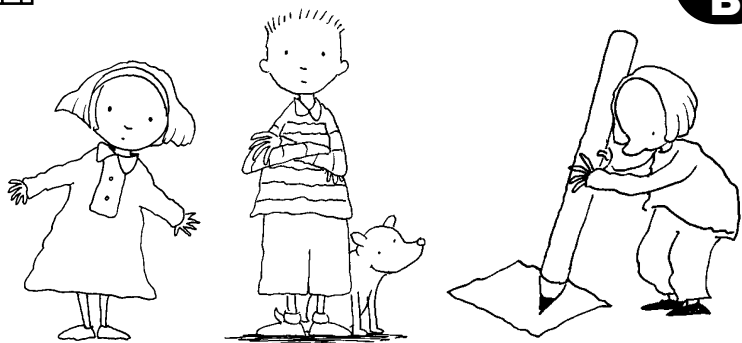


### Problem 63

Measures

Level B

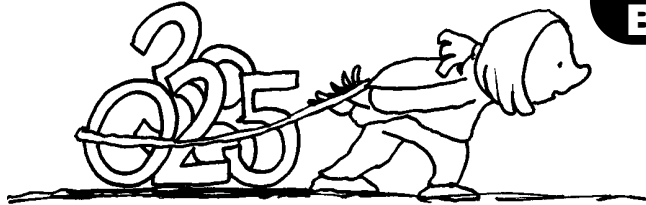
The average height of three people is 170 cm. If you are one of these people, who might be the other two people included in the average?



**Problem 64** Numbers **123**

Level  
**B**

Use zero and the numbers 2, 3, 5, 8, and 9 to make a sum that gives the greatest possible difference.



$$\begin{array}{r} \square \square \square \\ - \square \square \square \\ \hline \end{array}$$

(The larger number you make should be at the top. You can't use the zero in the hundreds place.)

**Problem 65** Numbers **123**

Level  
**B**

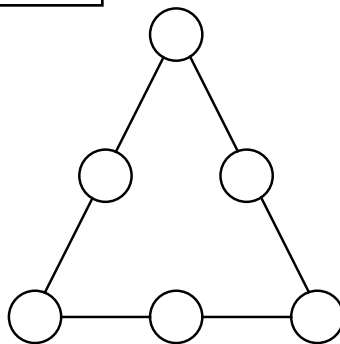
By using the digits from 0 to 9 in the algorithms below, find (a) the largest possible number and (b) the smallest possible number you can make. (You may not use the same digit more than once in any algorithm.)

$$\begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array} \quad \begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array} \quad \begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array} \quad \begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array}$$

**Problem 66** Numbers **123**

Level  
**B**

Using the even numbers 2, 4, 6, 8, 10 and 12, fill in the circles so that each side of the triangle adds to 20.



# PROBLEM SOLVING TASK CARDS -

## Open-Ended Problem Solving

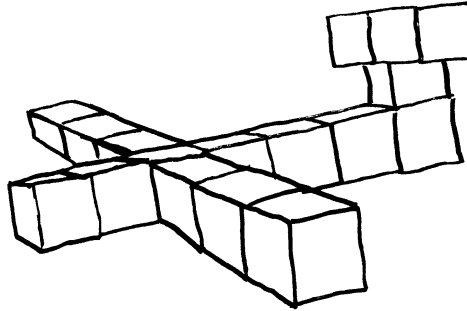
### Problem 67

Shape & Space

Measures

Level C

Take 24 centicubes. Build four different models using all the cubes. Draw them. Write a statement about the volume of each model.



### Problem 68

Measures

Level C

A rectangle has an area of  $24 \text{ cm}^2$ . What might be the length of its sides?



### Problem 69

Numbers 123

Level C

Write down fifteen numbers that could fall between  $7.3$  and  $7.4$ .



# PROBLEM SOLVING TASK CARDS -

## Open-Ended Problem Solving

### Problem 70

Numbers **123**

Level  
**C**

Find four digits to multiply that will have a product that is as close to 789 as possible. Use a calculator.

$$\begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array} \quad \begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array} \quad \begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array} \quad \begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array} \quad \begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array} \quad \begin{array}{r} \square \square \\ \times \square \square \\ \hline \end{array}$$

### Problem 71

Numbers **123**

Level  
**C**

Find at least five solutions, using counters numbered from 1 to 9. The total you are aiming for is 999.

$$\begin{array}{r} \square \square \square \\ + \square \square \square \\ \hline 9 \ 9 \ 9 \end{array} \quad \begin{array}{r} \square \square \square \\ + \square \square \square \\ \hline 9 \ 9 \ 9 \end{array} \quad \begin{array}{r} \square \square \square \\ + \square \square \square \\ \hline 9 \ 9 \ 9 \end{array}$$
  

$$\begin{array}{r} \square \square \square \\ + \square \square \square \\ \hline 9 \ 9 \ 9 \end{array} \quad \begin{array}{r} \square \square \square \\ + \square \square \square \\ \hline 9 \ 9 \ 9 \end{array} \quad \begin{array}{r} \square \square \square \\ + \square \square \square \\ \hline 9 \ 9 \ 9 \end{array}$$

### Problem 72

Numbers **123**

Level  
**C**

Solve each number sentence by replacing the gaps with the same number. (The number is different for each number sentence.)

$$[(\ \_ + \ \_) \times \ \_] - \ \_ = 28$$

$$(\ \_ + \ \_) \div \ \_ - \ \_ = 0$$

$$\ \_ + [(\ \_ - \ \_) + \ \_] = 6$$

$$(\ \_ + \ \_) \times \ \_ = 242$$

$$(\ \_ \times \ \_) + (\ \_ - \ \_) - \ \_ = 20$$

$$(\ \_ + \ \_) \times \ \_ = 450$$

# Answers to Task Cards Open-Ended Problem Solving

## Problem 55

A rectangle has four sides, four angles, straight sides, two diagonals, two axes of symmetry. All its angles are right angles. Opposite sides are equal. It is a 2D plane shape.

## Problem 56

Teacher to check all models. Pupils should be encourage to be creative when building their models.

## Problem 57

4 cm, 4 cm, 8 cm  
4 cm, 5 cm, 7 cm  
4 cm, 6 cm, 6 cm  
5 cm, 6 cm, 5 cm

## Problem 58

Some solutions could be:

$$3456 \times 2 = 6912$$

$$3470 \times 2 = 6940$$

$$3498 \times 2 = 6996$$

$$3501 \times 2 = 7002$$

Encourage pupils to make a list of other 'close' solutions.

## Problem 59

$$3 + 2 + 1 = 6$$

$$3 \times 2 = 6$$

$$3 \times 2 \times 1 = 6$$

$$6 \div (3 \times 1) = 2$$

$$6 \div (2 \times 1) = 3$$

$$6 \div (2 \times 3) = 1$$

$$6 - (2 + 3) = 1$$

## Problem 60

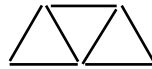
$$\begin{array}{r} 7 \boxed{8} 9 \\ + \boxed{5} 5 \boxed{6} \\ \hline \boxed{1} 3 4 5 \end{array}$$

There is only one solution.

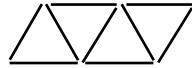
## Problem 61



5 toothpicks / 2 triangles



7 toothpicks / 3 triangles



9 toothpicks / 4 triangles



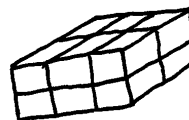
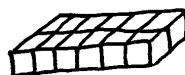
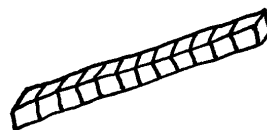
11 toothpicks / 5 triangles

As the odd number of toothpicks increases, a new triangle is added each time. For 21 toothpicks there will be  $(21 - 1) \div 2 = 10$  triangles.

If there is an even number of toothpicks, there will always be one side left over in this pattern.



## Problem 62



The volume is the same for each prism.

## Problem 63

In most cases, the pupil will be much shorter than 170 cm, so the other two people will have to be taller than 170 cm. Verify that the pupil worked out that the total heights would be 510 cm. It would be a valuable exercise to discuss the answers to this problem as a class.

# Answers to Task Cards Open-Ended Problem Solving

## Problem 64

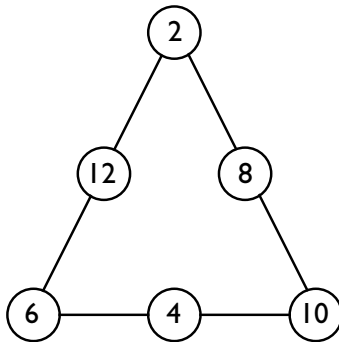
$$\begin{array}{r} 985 \\ - 203 \\ \hline 782 \end{array}$$

## Problem 65

$$\begin{array}{r} 96 \\ \times 87 \\ \hline 8352 \end{array}$$

$$\begin{array}{r} 23 \\ \times 10 \\ \hline 230 \end{array}$$

## Problem 66



## Problem 67

Teacher to check models. The volume of each model will be  $24 \text{ cm}^3$  as  $24 \times 1 \text{ cm}^3$  blocks were used.

## Problem 68

- 1 cm  $\times$  24 cm
- 2 cm  $\times$  12 cm
- 3 cm  $\times$  8 cm
- 4 cm  $\times$  6 cm
- 2.5 cm  $\times$  9.6 cm
- 2.4 cm  $\times$  10 cm
- 3.2 cm  $\times$  7.5 cm
- 1.5 cm  $\times$  16 cm

Complete list from pupils' answers.

## Problem 69

Teacher to check answers.

Any decimal number which starts with 7.3 and is followed by one or more digits is correct, e.g. 7.31, 7.3008.

## Problem 70

Some solutions are:

$$65 \times 12 = 780$$

$$17 \times 46 = 782$$

$$19 \times 42 = 798$$

$$21 \times 38 = 798$$

$$47 \times 17 = 799$$

## Problem 71

Any pairs using the combinations for 9 will work, e.g. 1 + 8, 2 + 7, 3 + 6, 4 + 5. Some possible solutions would be:

$$\begin{array}{r} 123 \\ + 876 \\ \hline 999 \end{array}$$

$$\begin{array}{r} 234 \\ + 765 \\ \hline 999 \end{array}$$

$$\begin{array}{r} 537 \\ + 462 \\ \hline 999 \end{array}$$

## Problem 72

$$[(4 + 4) \times 4] - 4 = 28$$

$$3 + [(3 - 3) + 3] = 6$$

$$(5 + 5) + (5 - 5) - 5 = 20$$

$$(2 + 2) \div 2 - 2 = 0$$

$$(11 + 11) \times 11 = 242$$

$$(15 + 15) \times 15 = 450$$

